Hilbert Schmidt stability and characters on amenable groups

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Stable systems of equations

Definition

For a matrix $A \in M_n(\mathbb{C})$, the ***normalized Hilbert Schmidt norm*** *is* $\|A\|_{HS,n}^2 = \frac{1}{n} Tr(A^*A) = \frac{1}{n}\sum_{i,j} |A_{i,j}|^2$

$R = \{ xyx^{-1}y^{-1} \}.$

Definition

Let $R \subset \mathbb{F}_k$ be a finite subset of the free group on k generators. We say the ***system of equations*** $\{\boldsymbol{r = 1} | \boldsymbol{r \in R}\}$ *is* ***HS-stable*** *if for any $\epsilon > 0$ there exists a $\delta > 0$ s.t. for any $n \in \mathbb{N}$ and any tuple $h_1, \ldots, h_k \in U(n)$ of unitaries with*

$$\forall r \in R \quad \|r(h_1, \ldots, h_k) - I_n\|_{HS,n} < \epsilon$$

There exists a solution $a_1, \ldots a_k \in U(n)$ with $\forall r \in R, \;\; r(a_1, \ldots, a_k) = I_n$ such that: $\|h_i - a_i\|_{HS,n} < \epsilon$ for all $i \leq k$.

Characters on groups & stability

Fact

Stability only depends on the group $\Gamma = {}^{\mathbb{F}_k}\!\diagup_{\langle\langle R\rangle\rangle}$.

Definition

A function $\tau : \Gamma \to \mathbb{C}$ *is called a* ***character*** *if it is the restriction of a trace on* $C^*(\Gamma)$ *to* Γ. τ *is* ***finite dimensional*** *if it is the trace of a finite dimensional representation.*

E.g. : $\mathbb{Z}^2$

Theorem (D. Hadwin & T. Shulman, 2018)

If $\Gamma = {}^{\mathbb{F}_k}\!\diagup_{\langle\langle R\rangle\rangle}$ *is amenable, then the following are equivalent:*

- *The system of equations* $\{r = 1 |\, r \in R\}$ *is HS-stable.*
- *Every character of* Γ *is a pointwise limit of finite dimensional characters.*

Thank you for listening!

References:

- Don Hadwin & Tatiana Shulman: Tracial stability for C^*-algebras. available at `https://arxiv.org/abs/1607.04470v2`
- Don Hadwin & Tatiana Shulman: Stability of group relations under small Hilbert-Schmidt perturbations. available at `https://arxiv.org/abs/1706.08405`