BMO spaces in σ-finite von Neumann algebras

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Von Neumann algebra types

- finite: exists n.f. tracial state

Von Neumann algebra types

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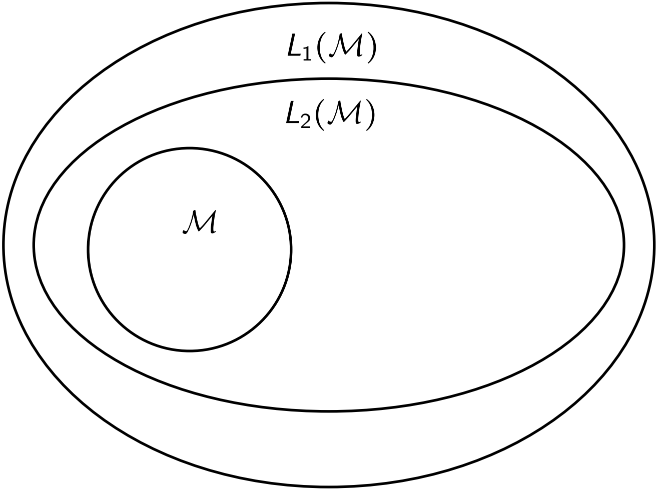
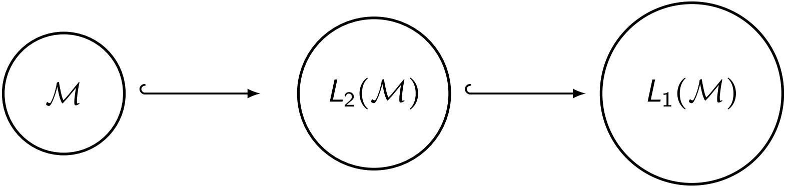
TUDelft

L_p-spaces for 'easy' case

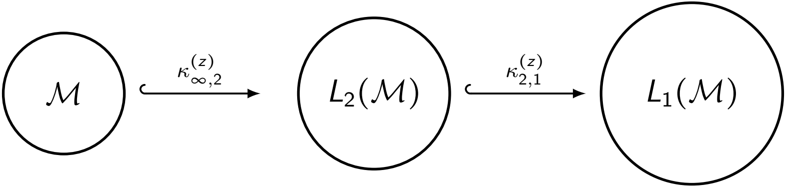


TUDelft

L_p-spaces 'difficult' case

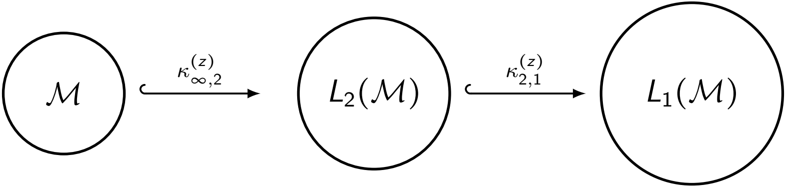


L_p-spaces 'difficult' case



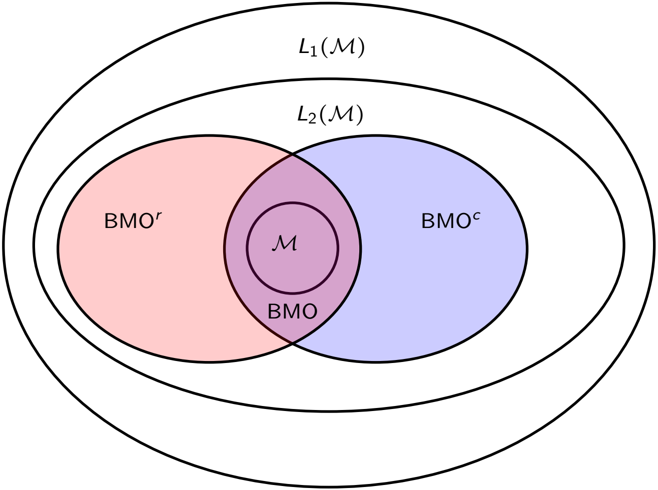
- $z \in [-1, 1]$

L_p-spaces 'difficult' case

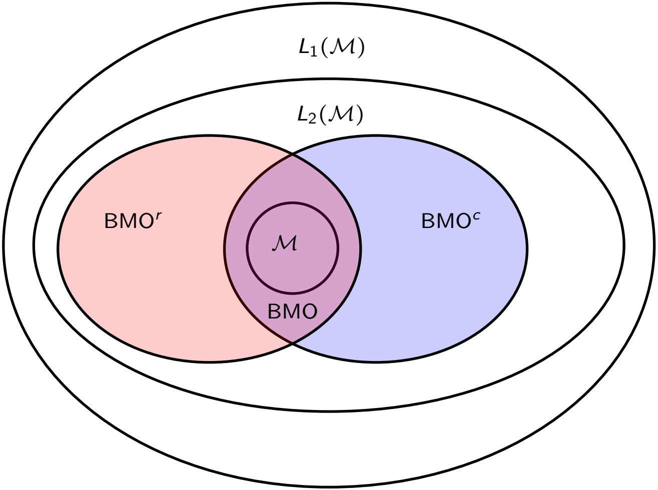


- $z \in [-1, 1]$
- $\kappa^{(z)}_{q,p}(x)^* = \kappa^{(-z)}_{q,p}(x^*)$

BMO 'easy' case



BMO 'easy' case



$$\|x\|_{\mathrm{BMO}} = \max\{\|x\|_{\mathrm{BMO}^c}, \|x\|_{\mathrm{BMO}^r}\}$$

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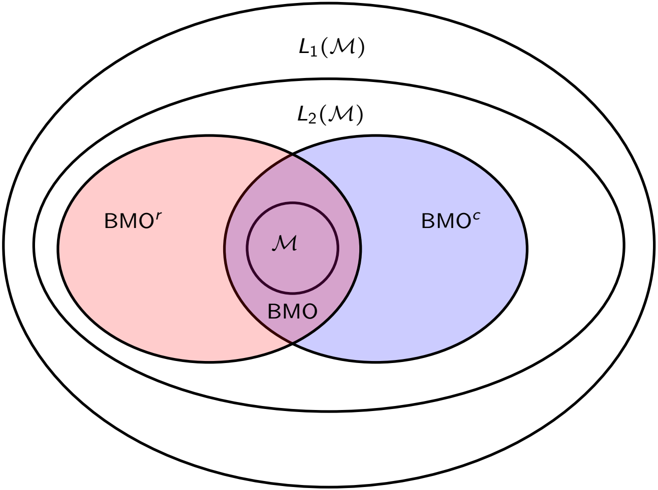

$$\|x\|_{\mathrm{BMO}} = \max\{\|x\|_{\mathrm{BMO}^c}, \|x\|_{\mathrm{BMO}^r}\} = \max\{\|x\|_{\mathrm{BMO}^c}, \|x^*\|_{\mathrm{BMO}^c}\}$$

BMO fact sheet

$$\|x\|_{\mathrm{BMO}^c} = \sup_{t\geq 0} \left\| \Phi_t\left(|x - \Phi_t(x)|^2\right) \right\|^{1/2}$$

$(\Phi_t)_{t\geq 0}$ 'nice' semigroup

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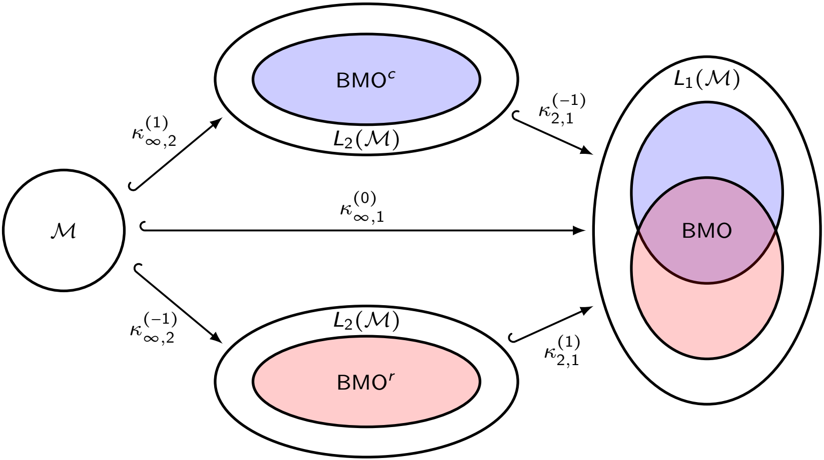
- $[\mathrm{BMO}, L_2]_{2/p} \cong L_p$
- Useful to prove L_p-boundedness.

BMO 'difficult' case



Result

Theorem (Caspers, V and Junge, Mei, Parcet)

BMO has a predual

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Corollary

BMO is a Banach space.