On the radial subalgebra for the quantum group O_F^+

Jacek Krajczok, IMPAN
joint work with Mateusz Wasilewski

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Let

- F_n be the free group on $2 \leqslant n < +\infty$ generators $g_1, \ldots, g_n$,
- $\mathcal{R} = \{(\lambda_{g_1} + \lambda_{g_1}^*) + \cdots + (\lambda_{g_n} + \lambda_{g_n}^*)\}''$ the radial subalgebra in $\mathrm{L}(F_n)$.

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- (Pytlik '81) $\mathcal{R}$ is a Maximal Abelian Subalgebra (MASA) in $\mathrm{L}(F_n)$.

Quantum group perspecitve

- F_n is a discrete quantum group and $\mathrm{L}(F_n) = \mathrm{L}^\infty(\widehat{F_n})$.
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- Let $\mathbb{G} = O_n^+$ be the free quantum orthogonal group (of Kac type).
- The analog of $\mathcal{R}$ is

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the algebra of class functions on O_n^+, where χ_1 is the character of the fundamental representation of O_n^+.

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the algebra of class functions on O_n^+, where χ_1 is the character of the fundamental representation of O_n^+.
- (Freslon, Vergnioux '16) $\mathscr{C}_{O_n^+}$ is MASA in $\mathrm{L}^\infty(O_n^+)$.

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- (Wasilewski, K. '21) No!

- Since $\sum_{\alpha\in\mathrm{Irr}(O_F^+)}\sqrt{\frac{\dim(\alpha)}{\dim_q(\alpha)}}<+\infty$, the inclusion $\mathscr{C}_{O_F^+}\subseteq \mathrm{L}^\infty(O_F^+)$ is quasi-split, i.e.

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$$\mathscr{C}_{O_F^+}\otimes_{alg}\mathrm{L}^\infty(O_F^+)^{op}\ni x\otimes y^{op}\mapsto xJ_hy^*J_h\in \mathrm{B}(\mathrm{L}^2(O_F^+))$$

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- (Bikram, Mukherjee '20) If $\mathscr{C}_{O_F^+}$ is MASA then $\mathrm{L}^\infty(O_F^+)\simeq \mathrm{B}(\ell^2)$, hence τ_t is inner and consequently trivial.

Thank you!