Amenable and Quasidiagonal traces - behaviour under homotopy

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Definition

(1) *A trace* $\tau : A \to \mathbb{C}$ *is called* *amenable* *if for all* $n \in \mathbb{N}$ *there is an integer* $k(n) \geq 1$ *and c.p.c. maps* $\phi_n : A \to \mathbb{M}_{k(n)}$ *such that*

$$\|\phi_n(ab) - \phi_n(a)\phi_n(b)\|_2 \to 0$$

and $tr_{k(n)}(\phi_n(a)) \to \tau(a)$ *for all* $a, b \in A$. *Here* $\|x\|_2 = tr_{k(n)}(x^*x)^{1/2}$.

(2) *A trace* τ *is* *quasidiagonal* *if for all* $n \in \mathbb{N}$ *there is an integer* $k(n) \geq 1$ *and c.p.c. maps* $\phi_n : A \to \mathbb{M}_{k(n)}$ *such that*

$$\|\phi_n(ab) - \phi_n(a)\phi_n(b)\| \to 0$$

and $tr_{k(n)}(\phi_n(a)) \to \tau(a)$ *for all* $a, b \in A$.

When amenability gives quasidiagonality

Proposition (Brown-Carrión-White)

Let A be a C^*-algebra. Then every amenable trace on $C_0(0,1] \otimes A$ is quasidiagonal.

Theorem (Tikuisis-White-Winter, Gabe)

Let A be a separable, exact C^*-algebra satisfying the UCT. Then every faithful amenable trace on A is quasidiagonal.

Is this a homotopy invariant property?

Proposition (N.)

Let A be a contractible C^*-algebra. Then all amenable traces on A are quasidiagonal.

Theorem (N.)

Let A be a separable exact C^*-algebra with a faithful amenable trace and such that A is homotopy equivalent to some other C^*-algebra B. If all amenable traces on B are quasidiagonal, then all amenable traces on A are quasidiagonal.