Morita Equivalence and Partial Actions

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Definition

Partial Action: Let A be a set. A partial action $\alpha = (\{A_t\}_{t\in G}, \{\alpha_t\}_{t\in G})$ on A, such that for every $t \in G$, $\alpha_t : A_{t^{-1}} \to A_t$ is an isomorphism. s.t.

- $A_1 = X$ and α_1 is the identity.
- $\alpha_t \circ \alpha_s \subseteq \alpha_{ts}$, for every $t, s \in G$.

C^***-Partial Action:**
Let A be a C^*-algebra. A partial action α, where for every $t \in G$, A_t is a closed two sided ideal of A and α_t is a $*$-isomorphism.
If $A_t = A$, for every $t \in G$, α is called a **global action**.

Definition

We will say that partial actions α^1 and α^2 acting on C^*-algebras $A^k, k = 1, 2$ are Morita-Rieffel equivalent if there exists a Hilbert $A^1 - A^2$-bimodule $\mathcal{M}$ and a (set theoretical) partial action $\gamma = (\{M_t\}_{t\in G}, \{\gamma_t\}_{t\in G})$ on $\mathcal{M}$ such that

- M_t is norm closed and sub-A^1A^2-bimodule of $\mathcal{M}$;
- $A_t^k = [\langle M_t, M_t\rangle_{A^k}]$;
- γ_t is a complex linear map;
- $\langle\gamma_t(\xi), \gamma_t(\eta)\rangle_{A^k} = \theta_t^k(\langle\xi, \eta\rangle_{A^k}$, for any $\xi, \eta \in M_{t^{-1}}$ and $k = 1, 2$.

Definitin

C*-globalization: Let α be a C*-partial action of G on C*-algebra A. A 4-tuple $(B, \beta, I, \mathfrak{i})$, where B is C*-algebra, β is a C*-global action of G on B, I is a C*-ideal of B and $\mathfrak{i} : \alpha \to \beta|_I$ is an isomorphism of C^*-partial actions.(i.e. $\mathfrak{i} = \{\mathfrak{i}_t\}_{t \in G}$, where for every $t \in G$, $\mathfrak{i}_t : A_t \to B \cap I$.)
A C*-globalization of C*-partial action α, is **minimal** if and only if

$$B = \overline{\Sigma_{t \in G} \alpha_t(A)}$$

Proposition

Let $\alpha = (\{A_t\}_{t \in G}, \{\alpha_t\}_{t \in G})$ be a partial action of group G on the C*-algebra A. Suppose that for $k = 1, 2$ minimal C*-globalization β^k acting on a C*-algebra B^k is given. Then there is an equivariant $*$-isomorphism

$$\phi : B^1 \to B^2$$

such that is the identity on the respective copies of A within B^1 and B^2.

Theorem

Every C*-partial action is Morita-Rieffel equivalent to one admitting a C^*-globalization. More precisely, every C*-algebraic partial action is Morita-Reiffel equivalent to the dual action Δ on the restricted smash product for the corresponding semi-direct product bundle (which admits a globalization).

Theorem

Let α and β be Morita-Rieffel equivalent

$$\alpha = (A, G, \{A_t\}_{t\in G}, \{\alpha_t\}_{t\in G}), \qquad \beta = (B, G, \{B_t\}_{t\in G}, \{\beta_t\}_{t\in G})$$

Then

- $A \rtimes_{red} G$ and $B \rtimes_{red} G$ are Morita-Rieffel equivalent.
- $A \rtimes G$ and $B \rtimes G$ are Morita-Rieffel equivalent.

Thank you for your attention!